



Sums and Differences to 1000  
Properties of Multiplication and Division  
Math in Focus

Unit 1 Curriculum Guide:  
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ORANGE PUBLIC SCHOOLS  
OFFICE OF CURRICULUM AND INSTRUCTION  
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# Unit Overview

## Unit 1: Chapters 1-5 & EUREKA Math Module (Properties of Multiplication and Division and Solving Problems with Units 2-5 and 10)

In this Unit Students will

- Round whole numbers to the nearest 10 or 100.
- Fluently add and subtract (with regrouping) two 2-digit whole numbers within 1000.
- Deconstruct word problems to determine the appropriate operation.
- Find the value of an unknown (expressed as a letter in an equation that is a representation of a two-step word problem and assess the reasonableness of the value.
- Use mental math strategies to add and subtract.

*Essential Questions*

- How is place value used to round numbers?
- How is place value used to add and subtract?
- How does the position of a digit in a number affect its value?
- In what ways can numbers be composed and decomposed?
- What are efficient methods for finding sums and differences?
- In what ways can items be grouped?
- What strategies can be used to make a reasonable estimate?
- How do units within a system relate to each other?

*Enduring Understandings*

- Numbers can be classified by attributes
- Numbers can represent quantity, position, location, and relationships
- Counting finds out the answer to “how many” in objects/sets
- Grouping (unitizing) is a way to count, measure, and estimate
- Standard units provide common language for communication measurements
- Understanding that place value is based on groups of ten (units of ten)
- Computation involves taking apart and combining numbers using a variety of approaches
- Flexible methods of computation involve grouping numbers in strategic ways
- Proficiency with basic facts aids estimation and computation of larger and smaller numbers
- Number patterns and relationships can be represented using variables
- Patterns can be generalized. Pattern can be found in many forms, grow, and repeat
- Mathematical expressions represent relationships

## MIF Pacing Guide Chapters 1-5

Activity	Common Core Standards	Estimated Time (# of block)	Lesson Notes
1.1 Counting	3. NBT 1	2 blocks	Students misinterpret the value of digits in multi-digit numbers. Frequently refer to a place value chart and connect the digits to conceptual models, i.e. Place value blocks and pictorial representations.
Lesson 1.3 Comparing and Ordering Numbers	3.OA.9	2 blocks	When students are comparing and ordering numbers have them to think about whether or not the pattern is increasing or decreasing. Also have students to observe whether a number pattern is increasing or decreasing by a particular place value
Chapter Test/ Performance Task	3.NBT.1, 3.NBT.2, and 3.OA.8	1 block	
Lesson 2.1/ 2.3 Mental Addition	3.NBT.2,	1-2 blocks	Teach mental math strategies that will encompass number bonds, number line, counting back/forward, compensation, bar models, and deconstructing. After these lessons, the strategies above should be included daily in some form (do now, math workstations, homework, and centers). See mental math strategies resources at end of guide.
Lesson 2.2 Mental Subtraction	3.NBT.2,	1-2 Blocks	
Lesson 2.4 Rounding Numbers to Estimate	3.NBT.1, 3.NBT.2, and 3.OA.8	2 blocks	Students should use a number line or base ten blocks to round numbers from 1 to 1,000.
Chapter Test/Performance Task	3.NBT.1, 3.NBT.2, and 3.OA.8	1 block	
Lesson 3.2 Addition with regrouping in hundreds with PS bar modeling	3.NBT.2 and 3.OA.8	1 block	Students may have a difficult time understanding how 10 ones or 10 of any unit becomes a new and greater unit.
Lesson 3.3 Addition with regrouping in Ones, Tens, and Hundreds with PS bar modeling	3.NBT.2 and 3.OA.8	1 block	
Chapter Test/ Performance Task	3.NBT.2, 3.MD.2 and 3.OA.8	1 block	
Lesson 4.1 Subtraction without regrouping with Problem Solving bar modeling	3.NBT.2 and 3.OA.8	1 block	Students do not demonstrate place value understanding.

Lesson 4.2 Subtraction with regrouping with Problem Solving bar modeling	3.NBT.2 and 3.OA.8	1 block	Students tend to subtract the small number from the larger number rather than regrouping. Ex $46-28=22$ . Instead of regrouping a ten as ten ones because it's not enough ones in the ones place in the number 46 to deduct 8 ones in the number 28, the student saw that the number on the 8 in 28 and took away 6, which is absolutely incorrect.
Lesson 4.3 Subtraction with regrouping with Problem Solving bar modeling	3.NBT.2 and 3.OA.8	1 block	They may struggle with breaking two-digit numbers into tens and ones.
Lesson 4.4 Subtraction across Zeros with Problem Solving bar modeling	3.NBT.2 and 3.OA.8	1 block	Students do not think about decomposing numbers into Tens and ones for easier adding and subtracting.
Chapter Test/Performance Task	3.NBT.2, and 3.OA.8	1 block	
Lesson 5.1 Real World Problems: Addition and Subtraction	3.NBT.2, and 3.OA.8	2 blocks	Students should work with bar models and how they can be used to represent the different word problem types in Table 1 from the CCSS.
Problem Solving	3.OA.8	1 block	

**EUREKA Pacing Guide**

**Math Module 1:**

**(Properties of Multiplication and Division and Solving Problems with units 2-5 and 10)**

Topic	Lesson	Lesson Objective/ Supportive Videos
<b>Topic A:</b> Multiplication and the Meaning of the Factors	Lesson 1	Understand <i>equal groups</i> of as multiplication. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 2	Relate multiplication to the array model. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 3	Interpret the meaning of factors – the size of the group or the number of groups. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic B:</b> Division as an Unknown Factor Problem	Lesson 4	Understand the meaning of the unknown as the size of the group in division. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 5	Understand the meaning of the unknown as the number of groups in division. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 6	Interpret the unknown in division using the array model. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>

<p><b>Topic C:</b> Multiplication Using Units of 2 and 3</p>	Lesson 7	Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 8	Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 9	Find related multiplication facts by adding and subtracting equal groups in array models. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 10	Model the distributive property with arrays to decompose units as a strategy to multiply. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<p><b>Topic D:</b> Division Using Units of 2 and 3</p>	Lesson 11	Model division as the unknown factor in multiplication using arrays and tape diagrams. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 12/13	Interpret the quotient as the number of groups or the number of objects in each group using units of 2 and 3. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a> <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<p><b>Topic E:</b> Multiplication and Division Using Units of 4</p>	Lesson 14	Skip-Count objects in models to build fluency with multiplication facts using units of 4. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 15	Relate arrays to tape diagrams to model the commutative property of multiplication. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 16	Use the distributive property as a strategy to find related multiplication facts. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 17	Model the relationship between multiplication and division. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>

<p><b>Topic F:</b></p> <p>Distributive Property and Problem Solving Using Units of 2-5 and 10</p>	Lesson 18-19	<p>Apply the distributive property to decompose units.</p> <p><a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a></p> <p><a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a></p>
	Lesson 20	<p>Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.</p> <p><a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a></p>
	Lesson 21	<p>Solve two-step word problems involving all four operations and assess the reasonableness of answers.</p> <p><a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a></p>
<p><b>End Of Module Assessment</b></p>		

## Common Core State Standards

**3.NBT.1**

Use place value understanding to round whole numbers to the nearest 10 or 100.

Students learn when and why to round numbers. They identify possible answers and halfway points. Then they narrow where the given number falls between the possible answers and halfway points. They also understand that by convention if a number is exactly at the halfway point of the two possible answers, the number is rounded.

Example: Mrs. Rutherford drives 158 miles on Saturday and 171 miles on Sunday. When she told her husband she estimated how many miles to the nearest 10 before adding the total. When she told her sister she estimated to the nearest 100 before adding the total. Which method provided a closer estimate?

**3.NBT.2**

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Problems should include both vertical horizontal forms, including opportunities for students to apply the commutative and associative properties. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable.

Example: There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the Playground?

Student 1	Student 2	Student 3	Student 4
$100 + 200 = 300$ $70 + 20 = 90$ $8 + 5 = 13$ $300 + 90 + 13 = 403$ students	I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403 students.	I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403 students.	$178 + 225 = ?$ $178 + 200 = 378$ $378 + 20 = 398$ $398 + 3 = 403$ students

**3.OA.1**

Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .*

- Students develop an initial understanding of multiplication of whole numbers by



modeling situations in which there are a specific number of groups with the same number of items in each group.

- Unlike addition, in which each addend represents a certain number of items, in multiplication one factor represents the number of groups and the other factor represents the number of items in each group. The product represents the total number of items in all of the groups.
- Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol ‘ $\times$ ’ means “groups of” and problems such as  $5 \times 7$  refer to 5 groups of 7.
- To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication, expression (e.g.,  $5 \times 6$ ) students interpret the expression using a multiplication context. They should begin to use the terms, *factor* and *product*, as they describe multiplication.

**For example:**

Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3,  $5 \times 3 = 15$ . Describe another situation where there would be 5 groups of 3 or  $5 \times 3$ . Sonya earns \$7 a week pulling weeds.

After 5 weeks of work, how much has Sonya worked? Write an equation and find the answer.

Describe another situation that would match  $7 \times 5$ .

**3.OA.2**

Interpret whole-number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .*

- Division can be understood by thinking in terms of finding a missing factor (either the number of groups or the number of items in a group)
- There are two distinct meanings of division.

**Partitive** (Fair sharing):

Knowing the total number of items product) and the number of groups (factor) to find the number of items in each group (missing factor).

**Measurement** ( Repeated Subtraction):

Knowing the total number of items (product) and the number of items in each group (factor) to find the amount in each group (missing factor)

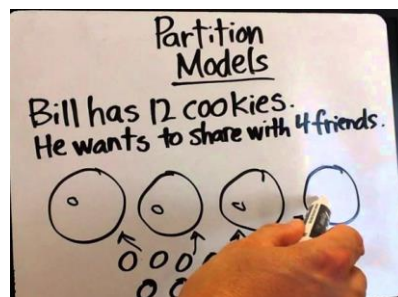
Type of Division	Number of Groups	Number of Items in group	Total number of Items
Partitive	Known	Unknown	Known
Measurement	Unknown	Known	Known

- Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor).
- To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g.,  $24 \div 6$ ) students interpret the expression in contexts that require both interpretations of division.

**For example:**

Partition models provide students with a total number and the number of groups. These models focus on the question, "How many objects are in each group so that the groups are equal?"

A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among 4 friends, how many cookies will each friend get?



**3.OA.3**

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

- Table 2 (below) provides problem situations for multiplication and division. These contexts provide important links to the developing conceptual understanding of the meaning of multiplication and division.
- Begin with modeling equal group situations and progress to array and area situations. Comparison situations do not need to be introduced until Grade 4.
- Students need many opportunities to use concrete materials to model the situations and identify the number of groups and the number of items in a group.
- Once students demonstrate understanding with multiplication situations, use connected division examples in which students identify the total number of objects and explain whether they know the number of groups or the number of items.

**3.OA.4**

Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations  $8 \times ? = 48$ ,  $5 = \square \div 3$ ,  $6 \times 6 = ?$ .*

- This standard is strongly connected to 3.OA.3 where students solve problems and determine unknowns in equations.
- Students should connect their understanding of modeling and explaining division situations to symbolic notation, writing equations.
- Focusing on the relationship between multiplication and division will help students develop fluency with related fact families.
- Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation.
- Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given  $4 \times ? = 40$ , they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

• Equations in the form of  $a \times b = c$  and  $c = a \times b$  should be used interchangeably, with the unknown in different positions.

**Examples:**

- Solve the equations below:  
 $24 = ? \times 6$   
 $72 \div \Delta = 9$
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether?  $3 \times 4 = m$

**3.OA.5**

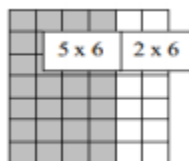
Apply properties of operations as strategies to multiply and divide.<sup>2</sup> *Examples: If  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known. (Commutative property of multiplication.)  $3 \times 5 \times 2$  can be found by  $3 \times 5 = 15$ , then  $15 \times 2 = 30$ , or by  $5 \times 2 = 10$ , then  $3 \times 10 = 30$ . (Associative property of multiplication.) Knowing that  $8 \times 5 = 40$  and  $8 \times 2 = 16$ , one can find  $8 \times 7$  as  $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ . (Distributive property.)*

- Properties should not be taught in isolation, but rather should be developed and discussed as part of student experiences. Incorporate opportunities for students to use the properties to develop strategies and patterns to simplify what is happening when they multiply two numbers.
- Identity Property: Multiplying a number by 1 does not change the number.
- Zero Property of Multiplication: If one of the factors is zero the product is zero.
- Commutative Property of Multiplication: Factors represent two different quantities- One factor represents the number of groups and the other factor represents the number of items in each group.

*Although  $6 \times 3$  and  $3 \times 6$  have the same product, the actual multiplication situations are not the same.*

- Associative Property of Multiplication: When multiplying three or more numbers, the product is always the same regardless of their grouping. This property is helpful in developing strategies for mental computation and decomposing factors to help students learn more difficult multiplication facts.  
Solving for the total number of items (the product) in “a groups with b items” ( $a \times b$ ) and c groups of these groups is the same as thinking about “a” groups of ( $b \times c$ ). Both ways of putting the groups of items together result in the same product because, regardless of how the groups are put together, the same number of items are being combined.
- Distributive property of Multiplication: Explored in the context of composing and decomposing factors. This will help students learn more difficulty basic facts.

**For example,** in the picture below the area of a  $7 \times 6$  figure can be determined by finding the area of a  $5 \times 6$  and  $2 \times 6$  and adding the two sums.



**3.OA.6**

Understand division as an unknown-factor problem. *For example, find  $32 \div 8$  by finding the number that makes 32 when multiplied by 8.*

- Multiplication and division are inverse operations and that understanding can be used to find the unknown.
- Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Example:

$$3 \times 5 = 15 \text{ \& } 5 \times 3 = 15$$

$$15 \div 3 = 5 \text{ \& } 15 \div 5 = 3$$

- Students understand that multiplication and division are inverse operations and that understanding can be used to find the unknown.
- Number Bonds demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Examples:

$$5 \times 9 = 45 \text{ \& } 9 \times 5 = 45$$

$$45 \div 5 = 9 \text{ \& } 45 \div 9 = 5$$



*Equations in the form of  $a \div b = c$  and  $c = a \div b$  need to be used interchangeably, with the unknown in different positions.*

**3.OA.7**

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

- This is a culminating standard to show the outcome of multiplication and division understanding in this domain and fluency within 100.
- By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts.

- Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

**Strategies students may use to attain fluency include:**

- Multiplication by zero and one
- Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
- Tens facts (relating to place value,  $5 \times 10$  is 5 tens or 50)
- Five facts (half of tens)
- Skip counting (counting groups of \_\_ and knowing how many groups have been counted)
- Square numbers (ex:  $3 \times 3$ )
- Nines (10 groups less one group, e.g.,  $9 \times 3$  is 10 groups of 3 minus one group of 3)
- Decomposing into known facts ( $6 \times 7$  is  $6 \times 6$  plus one more group of 6)
- Turn-around facts (Commutative Property)
- Fact families (Ex:  $6 \times 4 = 24$ ;  $24 \div 6 = 4$ ;  $24 \div 4 = 6$ ;  $4 \times 6 = 24$ )
- Missing factors

*General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal written forms.*

**3.OA.8**

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

- Students solve two-step problems that include more than one operation by representing the information using concrete models, pictures including bar models, and number lines. Writing equations begins with making connections between the representations and the symbolic notation (equations).
- Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use that make most sense them.

***Determining whether answers are reasonable by using number sense, understanding the context, the meaning of operations using mental computation strategies, and estimation strategies cannot be overemphasized as students work with all of their ideas embedded in this Standard.***

- Using a letter standing for the unknown quantity should explicitly connect to previous work with identifying missing information that was represented by a box, underscore, or other symbols.
- When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.

Example:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

**Student 1:**

I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I

**Student 2:**

I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is

**Student 3:**

I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30. I know my answer be about



get 500.	really close to 100. When I add that hundred to the 4 hundred that I already had, I end up with 500.	500.
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<b>3.OA.9</b>	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
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- Arithmetic patterns are patterns that change by the same rate, such as adding the same number.

**Example:**

The series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

- Teacher empowers students to examine and discover arithmetic patterns involving both addition and multiplication. Using a multiplication table, highlight a row of numbers and students record what they notice about the highlighted numbers. Through questioning, teacher coaches students in understanding and identifying patterns related to the properties of operations.

**Examples:**

Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends ( $14 = 7 + 7$ ).

Multiples of even numbers (2, 4, 6, and 8) are always even numbers. In an addition table, students examine patterns they notice.

**Discoveries should include:**

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

**Major Clusters**   **Supporting**   **Additional Clusters**

Common multiplication and division situations. <sup>1</sup>

	<b>UNKNOWN PRODUCT</b>	<b>GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)</b>	<b>NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)</b>
	<b><math>3 \times 6 = ?</math></b>	<b><math>3 \times ? = 18</math>, and <math>18 \div 3 = ?</math></b>	<b><math>? \times 6 = 18</math>, and <math>18 \div 6 = ?</math></b>
<b>EQUAL GROUPS</b>	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
<b>ARRAYS<sup>2</sup>, AREA<sup>3</sup></b>	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
<b>COMPARE</b>	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
<b>GENERAL</b>	<b><math>a \times b = ?</math></b>	<b><math>a \times ? = p</math> and <math>p \div a = ?</math></b>	<b><math>? \times b = p</math>, and <math>p \div b = ?</math></b>

<sup>1</sup> The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>2</sup> Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

<sup>3</sup> The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

# MIF Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	<p><b>Chapter Opener</b> Assessing Prior Knowledge</p> <p><i>The Pre Test serves as a diagnostic test of readiness of the upcoming chapter</i></p>	<p><b>Teacher Materials</b> Quick Check Pretest (Assessm't Bk) Recall Prior Knowledge</p> <p><b>Student Materials</b> Student Book (Quick Check); Copy of the Pre Test; Recall prior Knowledge</p>	<p>Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding</p> <p>Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test ; Students write Quick Check answers on a separate sheet of paper</p> <p>Quick Check and the Pre Test can be done in the same block (<i>See Anecdotal Checklist; Transition Guide</i>)</p> <p>Recall Prior Knowledge – Quick Check – Pre Test</p>
DIRECT ENGAGEMENT	<p><b>Direct Involvement/Engagement</b> Teach/Learn</p> <p><i>Students are directly involved in making sense, themselves, of the concepts – by interacting the tools, manipulatives, each other, and the questions</i></p>	<p><b>Teacher Edition</b> 5-minute warm up Teach; Anchor Task</p> <p><b>Technology</b> Digi</p> <p><b>Other</b> Fluency Practice</p>	<ul style="list-style-type: none"> <li>• The Warm Up activates prior knowledge for each new lesson</li> <li>• Student Books are CLOSED; Big Book is used in Gr. K</li> <li>• Teacher led; Whole group</li> <li>• Students use concrete manipulatives to explore concepts</li> <li>• A few select parts of the task are explicitly shown, but the majority is addressed through the hands-on, constructivist approach and questioning</li> <li>• Teacher facilitates; Students find the solution</li> </ul>
GUIDED LEARNING	<p><b>Guided Learning and Practice</b> Guided Learning</p>	<p><b>Teacher Edition</b> Learn</p> <p><b>Technology</b> Digi</p> <p><b>Student Book</b> Guided Learning Pages Hands-on Activity</p>	<p>Students-already in pairs /small, homogenous ability groups; Teacher circulates between groups; Teacher, anecdotally, captures student thinking</p> <p><b>Small Group w/Teacher circulating among groups</b> Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time</p>

INDEPENDENT PRACTICE	<p><b>Independent Practice</b></p> <p><i>A formal formative assessment</i></p>	<p><b>Teacher Edition</b> Let's Practice</p> <p><b>Student Book</b> Let's Practice</p> <p><b>Differentiation Options</b> All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment</p>	<p><b>Let's Practice</b> determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice.</p> <p>Manipulatives <b>CAN</b> be used as a communications tool as needed.</p> <p>Completely Independent</p> <p>On level/advance learners should finish all workbook pages.</p>
	<p><b>Extending the Lesson</b></p>	<p>Math Journal Problem of the Lesson Interactivities Games</p>	
ADDITIONAL PRACTICE	<p><b>Lesson Wrap Up</b></p>	<p>Problem of the Lesson</p> <p>Homework (Workbook, Reteach, or Extra Practice)</p>	<p>Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice)</p> <p>Reteach Homework (issued to struggling learners) should be checked the next day</p>
	<p><b>End of Chapter Wrap Up and Post Test</b></p>	<p><b>Teacher Edition</b> Chapter Review/Test Put on Your Thinking Cap</p> <p><b>Student Workbook</b> Put on Your Thinking Cap</p> <p><b>Assessment Book</b> Test Prep</p>	<p>Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is <u>graded/scored</u>.</p> <p>The Chapter Review/Test can be completed</p> <ul style="list-style-type: none"> <li>Individually (e.g. for homework) then reviewed in class</li> <li>As a 'mock test' done in class and doesn't count</li> <li>As a formal, in class review where teacher walks students through the questions</li> </ul> <p>Test Prep is completely independent; scored/graded</p> <p>Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own.</p>
POST TEST			

## Math Background

- During their elementary mathematics education, students were exposed to counting, reading and writing numbers up to 100 in Grade 2. Students have had countless exposure and practice with using Base-10 blocks to develop the association between the physical representation of the number, the symbol and number-word.
- Furthermore, students learned to add using vertical form where 10 ones or 10 tens were regrouped as a new unit of 1 ten or 1 hundred. Students were shown and given opportunities to demonstrate concrete representations with place-value charts and strips showing hundreds, tens and ones for numbers up to 100.
- Given a 3-digit number, students were expected to identify the place value of each digit in the whole number and express the number in standard, word and expanded form. Students frequently came up with their own algorithms to add, subtract, order, compare numbers and identify missing numbers in a pattern on and off a number line by applying place-value concepts.
- Students were held accountable for verbally communicating to each other and teacher by describing the differences between whole numbers using terms such as, least, fewest, less than, greater than, greatest, and equal to or have the same value as.

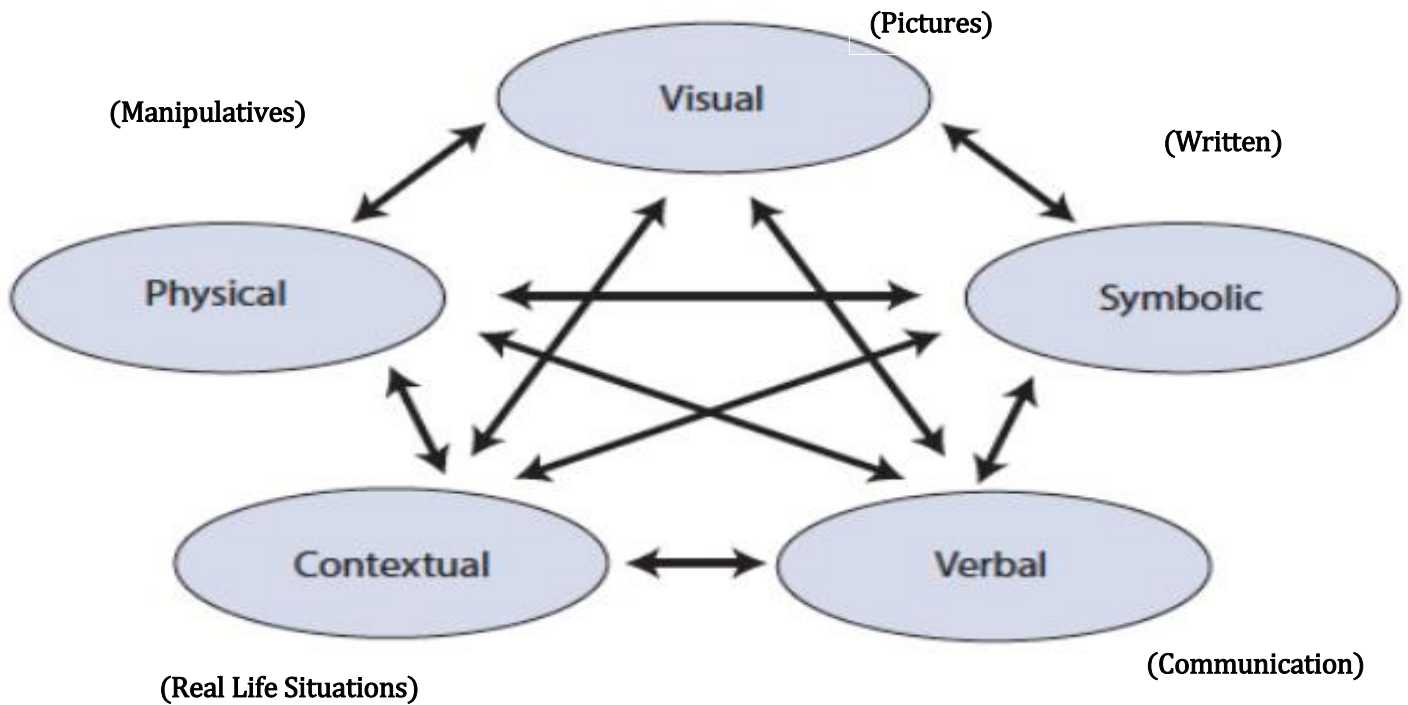
## Misconceptions

- Students misinterpret the value of digits in multi-digit numbers. Students need to understand that when you have ten of one unit, you also have one unit of the next higher value. Frequently refer to a place value chart and connect the digits to conceptual models, ie. place value blocks and pictorial representations. Have students create multiple ways to represent numbers such as 132 can be made of 1 hundred, 3 tens, 2 ones or 1 hundred, 1 ten, and 22 ones, or 12 tens and 12 ones. When explaining strategies used, students must identify the unit value; e.g when adding 492 and 265, they state that they are adding "two hundred" to "four hundred", ie. the 2 in 265 is named "two hundred", rather than "two".
- Students believe that subtraction is commutative. After students have discovered and applied the commutative property for addition, ask them to investigate whether this property works for subtraction. Have students share and discuss their reasoning and guide them to conclude that the commutative property does not apply to subtraction.
- Students misunderstand the meaning of the equal sign. The equal sign means "is the same quantity as" but many students believe the equal sign tells you that the "answer is coming up" to the right of the equal sign. Students need to see equations written multiply ways. It is important to model equations in various ways  $28 = 20 + 8$  or  $19 + 8 = 20 + 7$ .
- Students often get confused with naming value before understanding the amount of units represented.
- Students misunderstand the characteristics structure of our base-10 number system.(Place value and the position of a digit represent its value, base ten elements, which is based on the powers of ten that increases and decreases when shifted to the left or right, and is collections of ten which determine a new collection, the use of zero to show an absence value or to regroup numbers. Lastly additive structure which can be written in expanded notation.
- Students not being to apply place value to partitioning, comparing, ordering, rearranging, arithmetic patterns, and regrouping numbers up to 100 (Multi-unit counting)
- Students always subtract the small number from the larger number rather than regrouping. Ex  $46 - 28 = 22$ . Instead of regrouping a ten as ten ones because it's not enough ones in the ones place in the number 46 to deduct 8 ones in the number 28, the student saw that the number on the 8 in 28 and took away 6, which is absolutely incorrect.

## PARCC Assessment Evidence/Clarification Statements

CCSS	Evidence Statement	Clarification	Math Practices
3.OA.8-1	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	<p>i) Only the answer is required (methods, representations, etc. are not assessed here).</p> <p>iii) Addition, subtraction, multiplication, and division situations in these problems may involve any of the basic situations types with unknowns in various positions.</p> <p>iii) If scaffolded, one of the 2 parts must require 2-steps. The other part may consist of 1-step.</p> <p>iv) Conversions should be part of the 2-steps and should not be a step on its own.</p> <p>v) If the item is 2 points, the item should be a 2 point, un-scaffolded item but the rubric should allow for 2-1-0 points.</p>	1, 4
3.MD.2-1	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).		
3.MD.2-2	Add, subtract, multiply, or divide (this unit just add/subtract) to solve one step word problems involving masses or volumes that are given in same units, e.g. by using drawings (such as beakers with a measurement scale) to represent the problem.	i) Only the answer is required (methods, representations, etc. are not assessed here).	1,2,4,5
3.NBT.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	<p>i) Tasks have no context.</p> <p>ii) Tasks are not timed</p>	

## Use and Connection of Mathematical Representations



**The Lesh Translation Model**

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical:** The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal:** Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.



**Symbolic:** Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

### **The Lesh Translation Model: Importance of Connections**

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

## Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

**Concrete:** “Doing Stage”: Physical manipulation of objects to solve math problems.

**Pictorial:** “Seeing Stage”: Use of imaged to represent objects when solving math problems.

**Abstract:** “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

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## Read, Draw, Write Process

**READ** the problem. Read it over and over.... And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

## Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

### Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



The most  
important thing  
is to **NEVER**  
stop  
questioning

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

**100** questions that promote  
**Mathematical Discourse**

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** \_\_\_?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** \_\_\_ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is **mathematically correct**

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

## Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** \_\_\_\_\_?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

## Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

## Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?



## Help students learn to **conjecture, invent, and solve** problems

- 48 What would happen if \_\_\_?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram** or **make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.



## Help students learn to connect mathematics, its ideas, and its application

- 74 What is the **relationship** between \_\_\_ and \_\_\_?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?

- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to \_\_\_?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

### Help students persevere

- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?

### Help students focus on the mathematics from activities

## **Conceptual Understanding**

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

## **Procedural Fluency**

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.



## **Math Fact Fluency: Automaticity**

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the [mind](#) with the low-level details required, allowing it to become an automatic response pattern or [habit](#). It is usually the result of [learning](#), [repetition](#), and practice.

### **3-5 Math Fact Fluency Expectation**

**3.OA.C.7:** Single-digit products and quotients (Products from memory by end of Grade 3)

**3.NBT.A.2:** Add/subtract within 1000

**4.NBT.B.4:** Add/subtract within 1,000,000/ Use of Standard Algorithm

**5.NBT.B.5:** Multi-digit multiplication/ Use of Standard Algorithm

## Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

### Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

### Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

*Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*

(Wiliam 2007, pp. 1054; 1091)

## Unit 1 Assessment / Authentic Assessment Framework

Assessment	CCSS	Estimated Time	Format
Diagnostic Assessment (IREADY)		1-2 blocks	Individual
Chapter 1			
Optional Pre Test1	3.NBT.1	½ block	Individual
Optional Chapter 1 Test/ Performance Task	3.NBT.1	1 block	Individual
Authentic Assessment #1	3.NBT.1	½ block	Individual
Chapter 2			
Optional Pre Test 2	3.NBT.2	½ block	Individual
Optional Chapter 2 Test/ Performance Task	3.NBT.2	1 block	Individual
Authentic Assessment #2	2.OA.2	½ block	Individual
Chapter 3			
Optional Pre-Test 3	3.NBT.2	½ block	Individual
Optional Chapter 3 Test/ Performance Task	3.NBT.2	1 block	Individual
Authentic Assessment #3	3.NBT.2	1 block	Pairs
Chapter 4			
Optional Pre-Test 4	3.NBT.2	½ block	Individual
Optional Chapter 4 Test/ Performance Task	3.NBT.2	1 block	Individual
Chapter 5			
Optional Pre-Test 5	3.NBT.2, 3.OA.8	½ block	Individual
Optional Chapter 5 Test/ Performance Task	3.NBT.2, 3.OA.8	1 block	Indi
Eureka Module 1:	Properties of Multiplication and Division and Solving Problems with units 2-5 and 10.		
Optional End of Module Assessment	3.OA.1-8	1 block	Individual
Grade 3 Interim Assessment 1 (IREADY)	3.NBT.1-2 3.OA.8	1-2 Block	Individual

	PLD	Genesis Conversion
<b>Rubric Scoring</b>	PLD 5	100
	PLD 4	89
	PLD 3	79
	PLD 2	69
	PLD 1	59

## Connections to the Mathematical Practices

### **Student Friendly Connections to the Mathematical Practices**

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

## Connections to the Mathematical Practices

<b>1</b>	<p><b>Make sense of problems and persevere in solving them</b></p> <p>In <b>third</b> grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try approaches. They often will use another method to check their answers.</p>
<b>2</b>	<p><b>Reason abstractly and quantitatively</b></p> <p>In <b>third</b> grade, students should recognize that number represents a specific quantity. They connect quantity to written symbols and create logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.</p>
<b>3</b>	<p><b>Construct viable arguments and critique the reasoning of others</b></p> <p>In <b>third</b> grade, mathematically proficient students may construct viable arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, “How did you get that?” and “Why is it true?” They explain their thinking to others and respond to others’ thinking.</p>
<b>4</b>	<p><b>Model with mathematics</b></p> <p>Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language) drawing pictures, using objects, acting out, making chart, list, or graph, creating equations etc....Students need opportunities to connect different representations and explain the connections. They should be able to use all of the representations as needed. <b>Third</b> graders should evaluate their results in the context of the situation and reflect whether the results make any sense.</p>
<b>5</b>	<p><b>Use appropriate tools strategically</b></p> <p><b>Third</b> graders should consider all the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For example, they might use graph paper to find all possible rectangles with the given perimeter. They compile all possibilities into an organized list or a table, and determine whether they all have the possible rectangles.</p>
<b>6</b>	<p><b>Attend to precision</b></p>

	Mathematical proficient <b>third</b> graders develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying their units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle the record their answer in square units.
7	<b>Look for and make use of structure</b>
	In <b>third</b> grade, students should look closely to discover a pattern of structure. For example, students properties of operations as strategies to multiply and divide. (Commutative and distributive properties.
8	<b>Look for and express regularity in repeated reasoning</b>
	Mathematically proficient students in <b>third</b> grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property.

## Effective Mathematics Teaching Practices

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

**Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

**Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

**Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

## **5 Practices for Orchestrating Productive Mathematics Discussions**

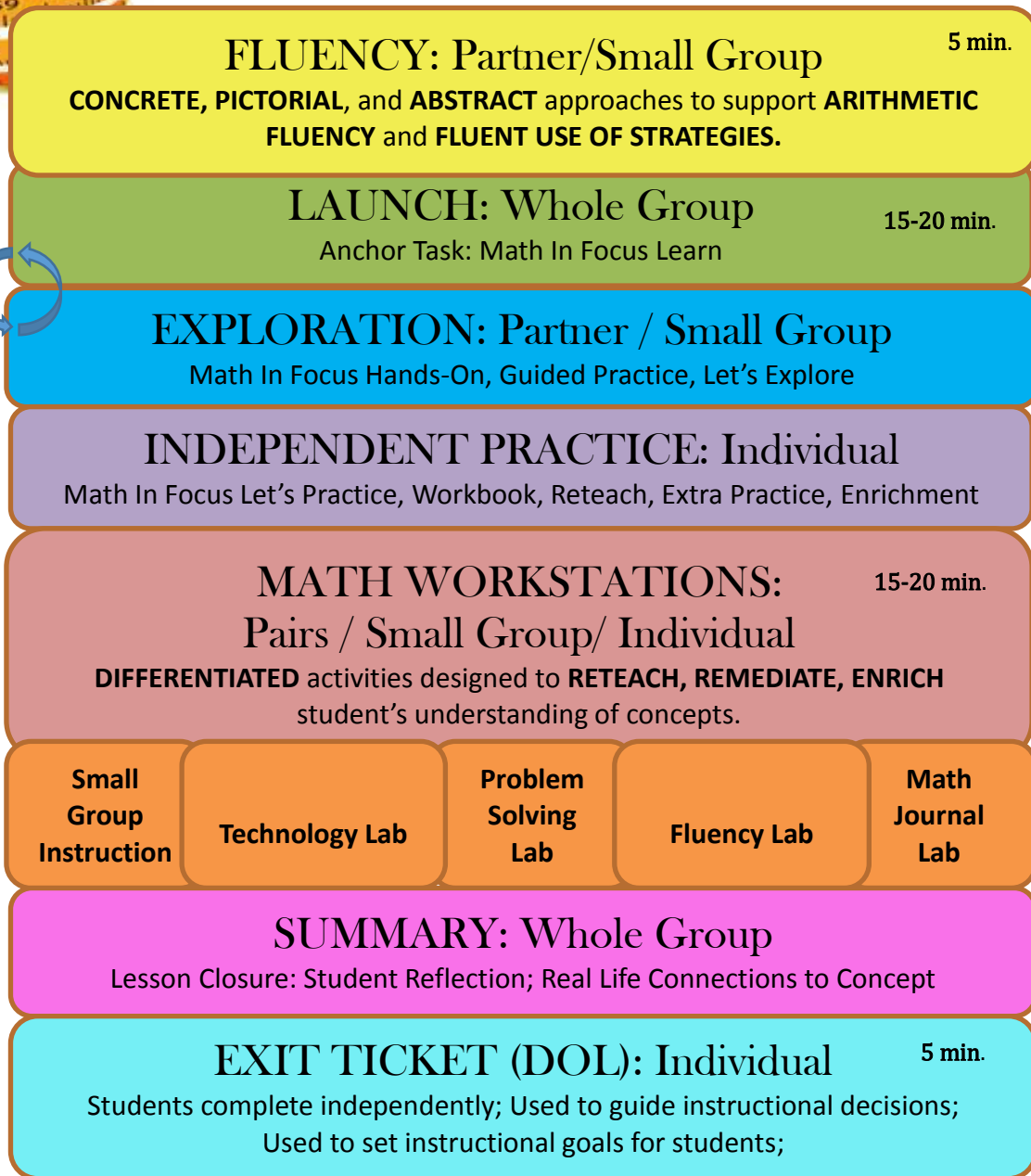
Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>





# 3<sup>rd</sup> and 4<sup>th</sup> Grade Ideal Math Block

## Essential Components



### Note:

- Place emphasis on the flow of the lesson in order to ensure the development of students' conceptual understanding.
- Outline each essential component within lesson plans.
- Math Workstations may be conducted in the beginning of the block in order to utilize additional support staff.
- Recommended: 5-10 technology devices for use within **TECHNOLOGY** and **FLUENCY** workstations.

## Authentic Assessment# 1

Name: \_\_\_\_\_

### Comparing Heights

Neil and Jerome were comparing their heights.



Neil measured his height and said,

***"I am 59 inches. 59 rounds to 100 so I am about 100 inches tall."***

Jerome measured his height and said,

***"I am 65 inches. 65 rounds to 70 so I am about 70 inches tall. You're taller, Neil."***

---

1. Is there something wrong with the boys' reasoning?
  
  
  
  
  
  
  
  
  
  
2. How could the boys correctly use rounding to compare their heights?
  
  
  
  
  
  
  
  
  
  
3. What are two examples of ways you could use rounding in your life?

## Authentic Assessment #1 Scoring Rubric

<b>Comparing Heights - 3.NBT.1</b>	
<b>Materials</b>	Comparing Heights handouts, paper, pencils, calculators (optional)
<b>Task</b>	<p>Distribute copies of the Comparing Heights handout.</p> <p>Read:</p> <ul style="list-style-type: none"> <li>• <i>Neil and Jerome were comparing their heights to see who is taller.</i></li> <li>• <i>Neil measured his height and said “I am 59 inches. 59 rounds to 100 so I am about 100 inches tall.”</i></li> <li>• <i>Jerome measured his height and said, “I am 65 inches. 65 rounds to 70 so I am about 70 inches tall. You’re taller, Neil.”</i></li> </ul> <p>Ask: 1. <i>What is wrong with the boys’ reasoning?</i></p> <p style="padding-left: 40px;">2. <i>How could the boys correctly use rounding to compare their heights?</i></p> <p style="padding-left: 40px;">3. <i>What are two examples of ways you could use rounding in your life?</i></p>

<b>Level 5: Distinguished Command</b>	<b>Level 4: Strong Command</b>	<b>Level 3: Moderate Command</b>	<b>Level 2: Partial Command</b>	<b>Level 1: No Command</b>
<p><b>Student gives all correct answers.</b></p> <p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction relationship</li> </ul> <p>Response includes an <b>efficient</b> and logical progression of steps.</p>	<p><b>Student gives all 3 correct answers.</b></p> <p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> <li>• relationship between multiplication and division</li> </ul> <p>Response includes a <b>logical</b> progression of steps</p>	<p><b>Student gives 2 correct answers.</b></p> <p>Constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> <li>• relationship between multiplication and division</li> </ul> <p>Response includes a <b>logical but incomplete</b> progression of steps. Minor calculation errors.</p>	<p><b>Student gives 1 correct answers.</b></p> <p>Constructs and communicates an incomplete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> <li>• relationship between multiplication and division</li> </ul> <p>Response includes an <b>incomplete or illogical</b> progression of steps.</p>	<p><b>Student gives less than 1 correct answers.</b></p> <p>The student shows no work or justification.</p>

## Authentic Assessment #2

Name: \_\_\_\_\_

Pencil and a Sticker

A pencil costs 59 cents, and a sticker costs 20 cents less. How much do a pencil and a sticker cost together?

## Authentic Assessment #2 Scoring Rubric:

**2.OA.1:** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Mathematical Practice:

**Type:** Individual, Individual w/Interview

**Materials:**

- Paper
- Pencil
- Hundreds board/chart
- Base Ten Blocks
- Student made number line

The purpose of the task is to allow children an opportunity to add with regrouping and subtract numbers. The solutions show how students can solve this problem before they have learned the traditional algorithm. Children need to be familiar with the 100s board, base ten blocks, counting on, and counting backwards. The solutions given make sense to children and are often easier for them to explain and justify than using the traditional algorithm.

Students who insist on using the standard algorithm should be able to clearly express each step they are making and why they are making it.

### Possible Solutions:

The Hundreds chart below represents the subtraction of  $59 - 20$ .

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The Hundred Chart below represents  $59 + 30 + 9$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

<b>SOLUTION:</b> $59 - 20 = 39$  $39 + 59 = 98$				
Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Student gives both correct answers.</p> <p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes an <b>efficient</b> and logical progression of steps.</p>	<p>Student gives both answers.</p> <p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes a <b>logical</b> progression of steps</p>	<p>Student gives one correct answer.</p> <p>Constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes a <b>logical but incomplete</b> progression of steps. Minor calculation errors.</p>	<p>Student gives one correct answer.</p> <p>Constructs and communicates an incomplete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes an <b>incomplete or illogical</b> progression of steps.</p>	<p>Student gives less than 1 answer.</p> <p>The student shows no work or justification.</p>

## Authentic Assessment #3

### Compatible Numbers

Name: \_\_\_\_\_

Look at Ms. Snyder's Game Board

<b>500</b>	<b>236</b>	<b>376</b>
<b>463</b>	<b>145</b>	<b>537</b>
<b>743</b>	<b>856</b>	<b>124</b>

- A. Ms. Snyder is playing a game with her class. In order to win round 1 of the game, the class must find **two** numbers on Ms. Snyder's game board whose sum is exactly 1,000. Which two numbers will win the game? Show all work.
- B. In order to win round 2 of the game, the class must find **three** numbers on Ms. Snyder's game board whose sum is exactly 1,000. Which three numbers will win the game? Show all work.
- C. With a partner assigned to you by your teacher, create your own game board that has a set of two numbers whose sum is exactly 1,000 and a set of three numbers whose sum is 1,000.

### Authentic Assessment 3 Scoring Rubric:

**3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

**Type:** Individual (Part A or B), individual with interview (Part A or B), and pairs (Part C)

The purpose of the task is to allow children an opportunity to add with regrouping and subtract numbers. The solutions show how students can solve this problem before they have learned the traditional algorithm. Children need to be familiar with the 100s board, base ten blocks, counting on, and counting backwards. The solutions given make sense to children and are often easier for them to explain and justify than using the traditional algorithm.

Students who insist on using the standard algorithm should be able to clearly express each step they are making and why they are making it.

<b>SOLUTION:</b>				
<ul style="list-style-type: none"> <li>• Student identifies that the sum of <b>463</b> and <b>537</b> is <b>1,000</b>.</li> <li>• Student identifies that the sum of <b>124</b>, <b>376</b>, and <b>500</b> is <b>1,000</b>.</li> <li>• Student clearly explains strategies for finding sums.</li> <li>• Students generates a game board with a set of two numbers whose sum is 1,000 and a set of three numbers whose sum is 1,000.</li> </ul>				
<b>Level 5: Distinguished Command</b>	<b>Level 4: Strong Command</b>	<b>Level 3: Moderate Command</b>	<b>Level 2: Partial Command</b>	<b>Level 1: No Command</b>
<p><b>Student gives all correct answers.</b></p> <p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes an <b>efficient</b> and logical progression of steps.</p>	<p><b>Student gives all correct answers.</b></p> <p>Clearly constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes a <b>logical</b> progression of steps</p>	<p><b>Student does 3 parts of the correct solution.</b></p> <p>Constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes a <b>logical but incomplete</b> progression of steps. Minor calculation errors.</p>	<p><b>Student does 1-2 parts of the correct solution.</b></p> <p>Constructs and communicates an incomplete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• properties of operations</li> <li>• relationship between addition and subtraction</li> </ul> <p>Response includes an <b>incomplete or illogical</b> progression of steps.</p>	<p><b>Student does not complete any part correct.</b></p> <p>The student shows no work or justification.</p>



# Additional Resources

## Mental Math Strategies

Thinking Strategies for Addition	
<p><b>Counting On:</b> Students start with a number and count on 1, 2, 3. For example, if the question is <math>5 + 2</math>, students count 5, 6, 7. Note: This strategy is only useful for adding 1, 2, or 3.</p>	<p><b>Using Doubles:</b> The first fact combinations students often learn are doubles. Examples:  <math>2 + 2 =</math>  <math>3 + 3 =</math>  <math>8 + 8 =</math></p>
<p><b>Making Ten:</b> Students make combinations that equal 10. Then they extend to make combinations that are multiples of 10. Examples: <math>6 + 4 = 10</math> extends to <math>76 + 4 = 80</math>. This can then be extended to <math>10 + 4 = 14</math> or <math>50 + 8 = 58</math>.</p>	
Thinking Strategies for Subtraction	
<p><b>Counting Back:</b> Students start with a number and count backwards. If the question is <math>5 - 2</math>, students count 5, 4, 3. Note: This strategy is only useful for subtracting 1, 2, or 3.</p>	<p><b>Counting Up:</b> Students start with a number being subtracted and count up to the number from which it is being subtracted. For example, for the question <math>9 - 7</math>, students can count 8, 9.</p>
<p><b>Using Part, Part, Whole:</b>            Given: Part + Part = Whole            Therefore: Whole – Part = Part</p> <p>Examples:            a. Thinking Addition:  <math>15 - 8 = ?</math>            Whole – Part = Part (?)            Students think <math>8 + 7 = 15</math> (Part + Part = Whole)            Therefore: <math>15 - 8 = 7</math></p>	<p>b. Partitioning:  <math>9 - 7 = ?</math>            Numbers include 9, 7, 2.            Students make all possible combinations for Part + Part = Whole  <math>7 + 2 = 9</math>  <math>2 + 7 = 9</math>            so <math>9 - 2 = 7</math> or <math>9 - 7 = 2</math></p> <p>c. Missing Part:  <math>8 + ? = 11</math>            Students use part, part, whole to answer such questions.</p>
When students have an easier time adding than subtracting the following strategies can be helpful.	
<p><b>Make Ten and Then Some:</b>            Given a subtraction question such as <math>14 - 8 = ?</math>, students start with the part (8), add-on to make 10 (i.e., <math>8 + 2</math>), then add-on from 10 to make 14 (<math>10 + 4</math>). Then the students add the numbers they added-on to make 14 (<math>4 + 2 = 6</math>).</p>	<p><b>Using Doubles:</b> For the question <math>13 - 6 = ?</math>, students think addition using doubles. For example, <math>6 + 6 = 12</math>, then add-on 1 to make 13, so <math>6 + 1 = 7</math>.</p>

<b>Thinking in Patterns</b>	
<p><i>Skip Counting:</i> Starting at any number, students skip count by 10s, 2s, 3s, 5s. For example, ask students to skip count by 10s starting at 46.</p>	<p><i>100 Chart:</i> Make sure a 100 chart is visible in your classroom and that students have access to desk-size charts. Refer to the chart and practice counting skills or the chart regularly.</p> <p><i>Arrow Moves:</i> Indicate moves on the 100 chart by using arrows. For example, <math>23 + 11 = ?</math>, would be indicated with one space across from 23 to 24 and then from 24 ten spaces down to 34. Note the pattern for all additions of +11 on the chart.</p>
<p><i>Chaining Operations:</i> Example: <math>8 + 2 + 4 + 6 - 3 = ?</math> (Note: choose combinations that end in multiples of 10 to encourage students' visualization of the 10 frame.)</p>	
<b>Strategies for Adding and Subtracting Large Numbers:</b>	
<p><b>Multiples of Ten:</b> For addition: <math>30 + 50 =</math> , <math>56 + 10 =</math> , <math>56 + 30 =</math> For subtraction: <math>50 - 30 =</math> , <math>56 - 10 =</math> , <math>56 - 30 =</math></p>	<p><b>Expanding the Second Addend or Subtrahend:</b> For addition: <math>28 + 17 =</math> , <math>28 + 10 + 7 =</math> For subtraction: <math>28 - 17 =</math> , <math>28 - 10 - 7 =</math></p>
<p><b>Front-end Adding:</b> Example: <math>65 + 26 = ?</math> Ask students to think <math>60 + 20 = 80</math> and <math>5 + 6 = 11</math>, so <math>80 + 11 = 91</math>.</p>	<p><b>Compensation for 8 and 9:</b> Examples: <math>67 - 19 = 67 - 20 + 1</math> <math>43 + 29 = 43 + 30 - 1</math> <math>67 - 18 = 67 - 20 + 2</math> <math>43 + 28 = 43 + 30 - 2</math></p>
<p><b>Compatible Numbers:</b> Students bring together numbers that add up to 10 or multiples of 10. Example: <math>8 + 5 + 12 + 7 + 5 + 3 + 4 = ?</math> Think <math>8 + 12 = 20</math>, <math>5 + 5 = 10</math>, <math>7 + 3 = 10</math> Therefore, <math>20 + 10 + 10 + 4 = 44</math></p>	<p><b>Multiples of 25:</b> Students count by 25s and relate to money.</p>
<p><b>Common Zeros:</b>For addition and subtraction operations, students remove the 0s, complete the operation, and then tack the 0 back on. Example: <math>120 - 70 = ?</math> Think <math>12 - 7 = 5</math> Add the <i>common zero</i>, so the answer is 50.</p>	

## **Problem Solving Questions Bank**

**1. We are in school 180 days. Imagine today is the 124th day of school. How many more days until we are out of school for summer vacation?**

**2. Roger needs a total of 23 box tops before he can win an awesome prize. So far he has collected 15 box tops. How many more does he need to reach his goal, and earn the prize?**

**3. Stephanie had bought a dozen eggs at the store. She now has 5 eggs left. How many eggs did she use?**

**4. This year Bob collect 134 pieces of candy when he went trick or treating. Two years ago he collected 87 pieces of candy. How many more did he collect this year?**

**5. Rosa read 57 pages of a book in the morning. She read 13 fewer pages in the afternoon. How many pages did Rosa read in the afternoon?**

**6. Mike has 57 action figures. Alex has 186 action figures. What would be the best estimate of the number of action figures Mike and Alex have altogether?**

**7. There are 500 sheets of paper in the pack Hannah bought. She has used 137 sheets already. How many sheets of paper does Hannah have left?**

**8. There were 378 visitors to the science museum on Friday. There were 409 visitors on Saturday. How many more people visited the museum on Saturday?**

**9. Jalen scores 247 points in a video game. How many more points does he need to score a total of 650?**

**10. Mia had 280 minutes to complete her chores. Mia spent 117 minutes washing the dishes. She spent 138 minutes cleaning her room. About how many minutes did she spend on the two tasks? Did Mia have enough time to watch a TV show that was 30 minutes?**

**11. Nathanael had 300 minutes to complete his book report before having to leave the house on Saturday. It took him 142 minutes to write the report. Then he spent 118 minutes correcting his report. About how much time did Nathanael spend on his report? Does Nathanael have enough time to read a book for 20 minutes?**

**12. After paying \$1,270 for a laptop, Mrs. Daniels has \$765 remaining. How much money did Mrs. Daniels have at first? Ray's rope is 1,452 centimeters long.**

**13. Hannah's rope is 379 centimeters longer than Ray's rope. A. How long is Hannah's rope? B. Ray uses 645 centimeters of his rope. How long is his remaining rope?**

**14. Chinaza has been collecting cards since she was 5 years old. She has not thrown away any of her cards. She collected 201 cards last year. She collects 125 cards this year. She has a total of 589 cards now. How many cards did she have in total at the end of last year? How many cards did she collect when she was 5 years old?**

**15. Na'Sean, Maurice, and Kygee hold a garage sale for their football team. Na'Sean raises \$350. Maurice raises \$20 more than Na'Sean. Kygee raises the same amount as the total amount raised by Na'Sean and Maurice. How much money do the three boys raise in all?**

**16. Savir has 120 baseball cards. After Savir bought 50 more cards, Savir had 35 more cards than Lucian. Write an equation you write to solve for Lucian's**

**baseball cards.**

**17. Samir wants to ride the Ferris wheel, the roller coaster, and the log ride. The Ferris wheel costs 3 tickets, the roller coaster costs 8 tickets and the log ride costs 7 tickets. Samir has 5 tickets. How many more tickets should Samir buy?**

**18. In Jamileth's desk drawer there are 11 yellow highlighters. There are 11 more pink highlighters than yellow highlighters, and there are 2 more blue highlighters than pink highlighters. How many highlighters are in Jamileth's desk drawer in all?**

**19. China needs 84 paper plates for a birthday party. She already has 14 blue plates and 30 red plates. How many more plates should China buy?**

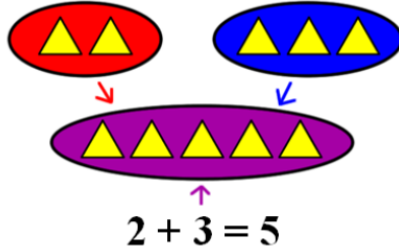
**20. Jamal needs 79 cupcakes for a birthday party. He already has 31 chocolate cupcakes and 20 vanilla cupcakes. How many more cupcakes should Jamal buy?**

## Visual Vocabulary

### Visual Definition

The terms below are for teacher reference only and are not to be memorized by students. Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

# add



To combine; put together two or more quantities.

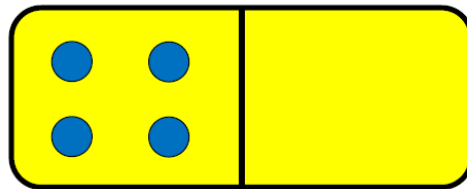
# addend

$$5 + 3 + 2 = 10$$

addends

Any number being added.

# Additive Identity Property of 0



$$4 + 0 = 4$$

When you add zero to a number, the sum is that same number.

# algorithm

$$\begin{array}{r} 47 \\ + 16 \\ \hline 13 \\ + 50 \\ \hline 63 \end{array}$$

Add the ones.  $7 + 6$   
Add the tens.  $40 + 10$   
Add the partial sums.

A step-by-step method for computing.

# Associative Property of Addition

$$(5 + 7) + 3 = 5 + (7 + 3)$$

$$12 + 3 = 5 + 10$$

$$15 = 15$$

Changing the grouping of three or more addends does not change the sum.

base-ten numeral form

12,345

3 is in the hundreds place.  
It has a value of  
3 hundreds or 300.

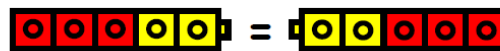
A common way of writing a number using digits. The value of a numeral depends on where it appears in the number. (also known as standard form)

base-ten numerals

0 1 2 3 4  
5 6 7 8 9

Any of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. The symbols can represent any amount based on a place value system of grouping by tens. (also known as digits)

# Commutative Property of Addition

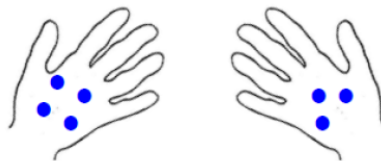


$$3 + 2 = 2 + 3$$

$$a + b = b + a$$

Changing the order of the addends does not change the sum.

compare



4 is more than 3

To decide if one number is greater than, less than, or equal to another number.

**compatible numbers**

$$\begin{array}{r} 57 \rightarrow 60 \\ + 23 \rightarrow + 25 \end{array}$$

Numbers that are easy to compute mentally and are close in value to the actual numbers. Compatible numbers can be used when estimating.

**compose**

$$\begin{array}{c} 300 + 40 + 2 \\ \swarrow \quad \downarrow \quad \searrow \\ 342 \end{array}$$

To put together smaller numbers to make larger numbers.

**counting number**



A whole number that can be used to count a set of objects. Counting numbers do not include 0. (e.g., 1, 2, 3, 4...)

**decompose**

$$\begin{array}{c} 342 \\ \swarrow \quad \downarrow \quad \searrow \\ 300 + 40 + 2 \end{array}$$

To separate a number into 2 or more parts.

**difference**

$$289 - 146 = 143$$

difference

The amount that remains after one quantity is subtracted from another.

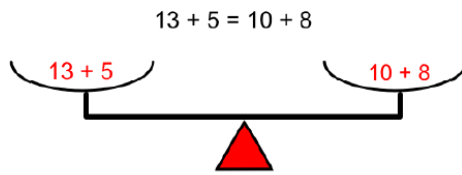
**digit**

0 1 2 3 4  
5 6 7 8 9

Any of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. (also known as base-ten numerals)



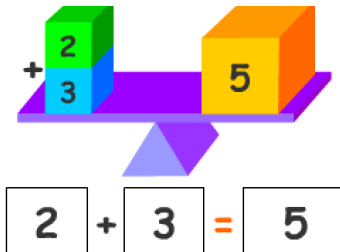
**equal**



Having the same value.

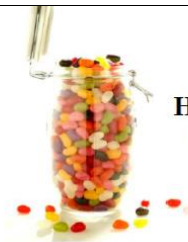
**equation**

These expressions balance the scale because they are equal.



A mathematical sentence with an equal sign. The amount on one side of the equal sign has the same value as the amount on the other side.

**estimate**



How many jelly beans are in the jar?

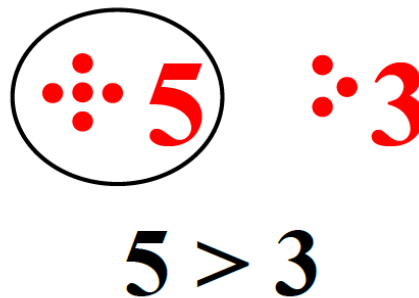
To find a number close to an exact amount; an estimate tells *about* how much or *about* how many.

**expanded form**

$$263 = 200 + 60 + 3$$

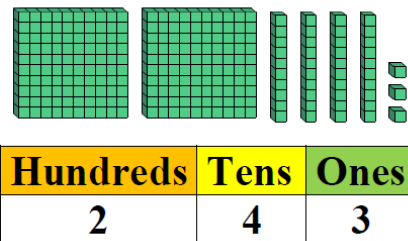
A way to write numbers that shows the place value of each digit.

**greater than**



Greater than is used to compare two numbers when the first number is larger than the second number.

**hundreds**



The value of a digit that is the third position from the right when describing whole number place value.

# kilogram (kg)



Math book

About  $2\frac{1}{2}$  pounds

A metric unit of mass equal to 1000 grams.

# less than



$$3 < 5$$

Less than is used to compare two numbers when the first number is smaller than the second number.

# number line



A diagram that represents numbers as points on a line.

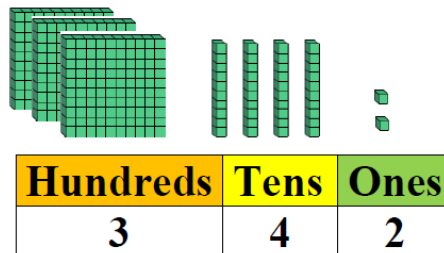
# ones



8 ones

A single unit or object.

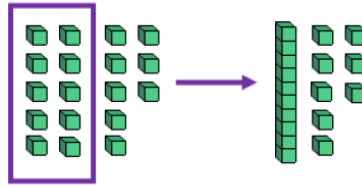
# place value



$$300 + 40 + 2$$

The value a digit has because of its place in a number.

# regroup



Regroup 18 ones as 1 ten and 8 ones.

To rearrange the formation of a group.

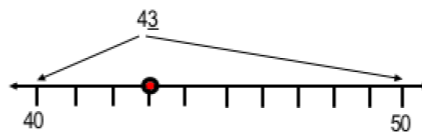
# related facts

## Related Facts for 3, 5, 8

$$\begin{array}{ll} 3 + 5 = 8 & 8 - 5 = 3 \\ 5 + 3 = 8 & 8 - 3 = 5 \end{array}$$

Related addition and subtraction facts or related multiplication and division facts. (also known as fact family)

# round a whole number



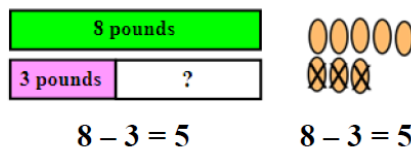
To find the nearest ten, hundred, thousand, (and so on).

# standard form

# 12,345

The common or usual way of writing a number using digits. (also known as base-ten numeral form)

# subtract



An operation that gives the difference between two numbers. Subtraction can be used to compare two numbers, or to find out how much is left after some is taken away.

$$453 + 929 = 1,382$$

**sum**

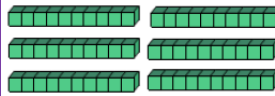
The answer to an addition problem.



**sum**  
Example

**tens**

3 x 20  
3 x 2 tens  
6 tens



**60**

Sets of ten ones.  
(i.e., 10, 20, 30, 40, 50, 60, 70, 80, or 90)

**variable**

$$5 \times b = 10$$

*b* is a variable worth 2

A letter or symbol that represents a number.

**whole numbers**



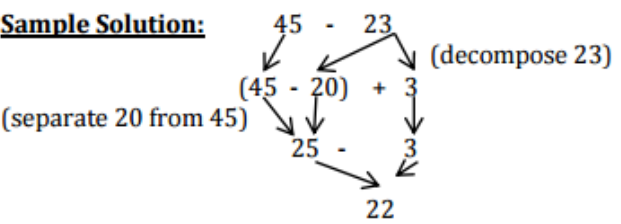
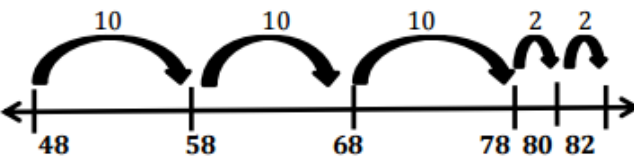
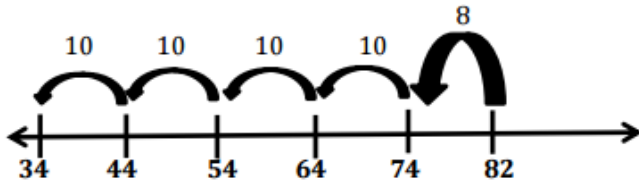
Whole numbers are 0 and the counting numbers 1, 2, 3, 4, 5, 6, and so on.

**word form**

The word form of  
**345**  
is three hundred  
forty-five.

A way of using words to write a number.  
(also known as number names)

## Teaching Multiple Representations

<p style="text-align: center;"><b>Counting Back</b></p> <p><b>Question:</b> 8-3</p> <p><b>Sample Solution:</b></p> <p>For counting back students would start at 8 and count backward 3 until they arrived at 5.</p> <p style="text-align: center;">8...7, 6, 5</p>	<p style="text-align: center;"><b>Removal in Parts</b></p> <p><b>Question:</b> 45 - 23</p> <p><b>Sample Solution:</b></p> 				
<p style="text-align: center;"><b>Constant Difference</b></p> <p><b>Question:</b> 57-22</p> <p><b>Sample Solution:</b></p> <p>Add 3 to each number and the difference remains the same. Only the numbers become friendlier to work with.</p> <p>57 - 22  <math>+3 \quad +3</math> (add 3 to each # keeps difference the same)          60 - 25          60-25=35</p>	<p style="text-align: center;"><b>Adding Up to find the Difference</b></p> <p><b>Question:</b> 82-48</p> <p><b>Sample Solution:</b> 82-48</p> <p style="text-align: center;"><math>48 + (10 + 10 + 10 + 4) = 82</math></p>  <p>Student adds up from 48 to 82 to find the difference of 34.</p>				
<p style="text-align: center;"><b>Part Whole Box Model</b></p> <p><b>Question:</b> 57-22</p> <p><b>Sample Solution:</b></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tbody> <tr> <td colspan="2" style="padding: 5px;">Whole 57</td> </tr> <tr> <td style="padding: 5px;">Part 22</td> <td style="padding: 5px;">Part 35</td> </tr> </tbody> </table> <p>Students understand the whole and one part of the whole. Because of this, the student is able to identify the other missing part of the whole.</p>	Whole 57		Part 22	Part 35	<p style="text-align: center;"><b>Adjusting 1 Number To Create An Easier Number</b></p> <p><b>Question:</b> 39 - 24</p> <p><b>Sample Solution:</b></p> <p style="text-align: center;">Adding one to 39 to make it a 40</p> <p style="text-align: center;"> <math>(39 (+1)) + 24</math>  <math>(40) - 24 = 16</math>  <math>16 (-1) = 15</math> </p> <p style="text-align: center;">Added 1 to 39 so 1 was removed from the sum</p>
Whole 57					
Part 22	Part 35				
<p style="text-align: center;"><b>Using a Number Line</b></p> <p><b>Question:</b> 82-48</p> <p><b>Sample Solution:</b> 82-48</p>  <p>Student adds up from 48 to 82 to find the difference</p>					

\*\*\*These strategies should be discovered, explored, and modeled by the students\*\*\*

# 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see [21<sup>st</sup> Century Career Ready Practices](#) .

# Resources

## **Think Central**

<https://www-k6.thinkcentral.com/ePC/start.do>

## **Common Core Tools**

<http://commoncoretools.me/>

<http://www.ccsstoolbox.com/>

<http://www.achievethecore.org/steal-these-tools>

## **Achieve the Core**

<http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12>

## **Manipulatives**

<http://nlvm.usu.edu/en/nav/vlibrary.html>

<http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000>

<http://www.thinkingblocks.com/>

## **Website Resources**

### **IllustrativeMath Project**

<http://illustrativemathematics.org/standards/k8>

The site contains sets of tasks that illustrate the expectations of various CCSS in grades K–8 grade and high school. More tasks will be appearing over the coming weeks. Eventually the sets of tasks will include elaborated teaching tasks with detailed information about using them for instructional purposes, rubrics, and student work.

### **Inside Mathematics**

<http://www.insidemathematics.org/index.php/tools-for-teachers>

Inside Mathematics showcases multiple ways for educators to begin to transform their teaching practices. On this site, educators can find materials and tasks developed by grade level and content area.

### **Engage NY**

[http://www.engageny.org/video-library?f\[0\]=im\\_field\\_subject%3A19](http://www.engageny.org/video-library?f[0]=im_field_subject%3A19)

### **IXL**

<http://www.ixl.com/>

### **Sample Balance Math Tasks**

<http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/>

### **Georgia Department of Education**

<https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx>

Georgia State Educator have created common core aligned units of study to support schools as they implement the Common Core State Standards.

3<sup>rd</sup>Grade:<http://ccgpsmathematicsk-5.wikispaces.com/3rd+Grade>

Formative Assessment :<http://ccgpsmathematicsk-5.wikispaces.com/K-5+Formative+Assessment+Lessons+%28FALs%29>

Number Talks and Multi-grade Resources: <http://ccgpsmathematicsk-5.wikispaces.com/Number+Talks+and+other+Multi+Grade+Resources>

### **NY SAMPLE QUESTIONS**

Grade 3: <https://docs.google.com/file/d/0Byj6JhSTYWXwb1F4aFc4MGNpWGM/preview>

### **Howard County**

3<sup>rd</sup>Grade :<https://grade3commoncoremath.wikispaces.hcpss.org/home>

### **Ohio**

[http://education.ohio.gov/getattachment/Topics/Ohio-s-New-Learning-Standards/Mathematics/Grade\\_3\\_Math\\_Model\\_Curriculum\\_March2015.pdf.aspx](http://education.ohio.gov/getattachment/Topics/Ohio-s-New-Learning-Standards/Mathematics/Grade_3_Math_Model_Curriculum_March2015.pdf.aspx)

### **Gates Foundations Tasks**

<http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf>

### **Minnesota STEM Teachers' Center**

<http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships>

### **Singapore Math Tests K-12**

<http://www.misskoh.com>

### **Math Score:**

Math practices and assessments online developed by MIT graduates.

<http://www.mathscore.com/>

### **Massachusetts Comprehensive Assessment System**

[www.doe.mass.edu/mcas/search](http://www.doe.mass.edu/mcas/search)

### **Performance Assessment Links in Math(PALM)**

PALM is currently being developed as an on-line, standards-based, resource bank of mathematics performance assessment tasks indexed via the National Council of Teachers of Mathematics (NCTM).

<http://palm.sri.com/>

### **Mathematics Vision Project**

<http://www.mathematicsvisionproject.org/>



## **NCTM**

<http://illuminations.nctm.org/>

## **Assessment Resources**

- \*Illustrative Math: <http://illustrativemathematics.org/>
- \*PARCC: <http://www.parcconline.org/samples/item-task-prototypes>
- NJDOE: <http://www.state.nj.us/education/modelcurriculum/math/>  
(username: model; password: curriculum)
- DANA: [http://www.ccsstoolbox.com/parcc/PARCCPrototype\\_main.html](http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html)
- New York: <http://www.p12.nysed.gov/assessment/common-core-sample-questions/>
- \*Delaware: <http://www.doe.k12.de.us/assessment/CCSS-comparison-docs.shtml>